Anisotropies in diffusion-limited aggregates on square-lattice diffusion-limited aggregates

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Diffusion-limited aggregates (DLA's) are grown on square-lattice DLA's in order to see such an anisotropy in small-scaled clusters as seen in large square-lattice DLA's and in order to probe square-lattice DLA's with random walks. To find physical origins of the anisotropy, we have grown such DLA's by different growth rules in which hopping directions of random walks and the choice of growth sites are varied.

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Even though diffusion-limited aggregation (DLA) [1] has successfully explained a broad range of nonequilibrium growths and aggregation process [2–5], the complete theoretical understanding of DLA has not been achieved. One of the main controversies on DLA is the strong dependence of the asymptotic structure of DLA on the underlying lattice structure [6–12]. The cluster shape of DLA on continuous media with the fractal dimension $D\simeq 1.7$ is nearly isotropic [4]. But on a square lattice, the envelopes of DLA clusters containing about 10^6 particles are cross shaped and D of such DLA's is about 1.55 [7,8]. To understand the anisotropy of a large square-lattice DLA's (SDLA's) several studies have been done by varying deposition rules [9,10] or by deterministic growths [11]. To see this anisotropy at small-scaled clusters and to understand the physical origin of the anisotropy, noise-reduced DLA's (or anisotropy-enhanced DLA's) [5,13–15] have been studied on two-dimensional lattices and it has been found that the anisotropy becomes more enhanced as the noise reduction parameter m increases. These results on SDLA suggest that there is a competition between the anisotropy provided by the underlying lattice and the randomness due to stochastic motions of incoming particles [2].

The object of this study is to grow DLA clusters of a small scale on a SDLA cluster [double DLA (DDLA) clusters] by using several different growth rules in order to see what make the clusters anisotropic or to find out which part of the growth algorithm is crucial to anisotropic DDLA clusters. DDLA clusters are grown by repeating the following four steps. (i) We put the seed on the seed site of an SDLA cluster. (ii) A particle which starts from a point on a starting circle continues *a certain kind of random walker's* (RW's) on a square lattice until the particle reaches *a growth site* of the DDLA cluster. (iii) If the growth site is occupied by a particle of the SDLA cluster, then the particle is attached to the DDLA cluster. (iv) Otherwise, the particle is abandoned and a new particle starts. We believe that this process is equivalent to *a probe of SDLA with a given kind of* RW's, because during the growth of a DDLA cluster, particles which do RW's of a chosen kind on a square lattice, for deposition (or being attached), *look for the growth sites which have already been occupied by particles of the SDLA cluster.*

To apply our growth rules we should choose the kind of RW's and growth sites. In an ordinary DLA model [4] the hopping directions of RW's on a square lattice are constrained to those along the four bonds which connect the site where the random walker is and four nearest neighbors (NN's) [Fig. (a)]. To know the effect of hopping directions of RW's on the structure of a cluster we also consider hopping directions along diagonals of fundamental plaquettes on a square lattice which connect the site where the random walker is and four next nearest neighbors (NNN's) [Fig. 1(b)]. In an ordinary DLA model only vacant NN's of a site in the cluster are allowed to be growth sites [Fig. 1(c)]. To know the effects of the choice of growth sites on the structure of a cluster, we also allow vacant NNN's of a site in the cluster as growth sites [Fig. 1(d)]. All SDLA clusters in this paper are assumed to be grown by the normal growth rule which is based on the combination of RW's in Fig. 1(a) and the deposition rule in Fig. 1(c): [(ac) rule]. However we grow DDLA clusters on SDLA clusters by several different growth rules. Such growth rules are as follows. The first and most important growth rule is of course the (ac) rule. The second rule is based on a combination of RW's in Fig. 1(a) and both NN's and NNN's of any site in a cluster as chosen growth sites [(acd) rule]. The third rule is

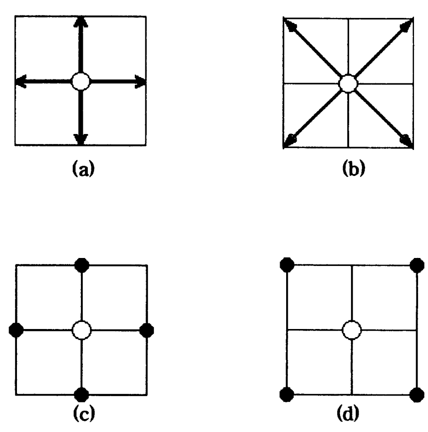

FIG. 1. (a) RW's with the directions of hopping along lattice bonds (unfilled circle denotes a site where the random walker is). (b) RW's with the directions of hopping along the diagonals of a plaquette of a square lattice. (c) NN's (filled circles) to a site in DDLA clusters (unfilled circle). (d) NNN's (filled circles) to a site in DDLA clusters.

based on a combination of RW's in Fig. 1(b) and only NN's of any site in a cluster as chosen growth sites [(bc) rule]. The final rule is based on a combination RW's in Fig. 1(b) and both NN's and NNN's of any site in a cluster as chosen growth sites [(bcd) rule].

If the anisotropy of SDLA clusters is due to a mechanism which makes the growth of clusters along the axes of the square lattice easier than along the diagonal directions, we expect that (ac) rule with steps (i)–(iv) make DDLA clusters grow along the axes more strongly than SDLA clusters themselves and we believe that such an anisotropy can be seen at a much smaller scale than the scale at which SDLA clusters show the anisotropy. The algorithm for the growth of DDLA clusters can be applied repeatedly and hierarchically as one can grow DLA clusters on DDLA clusters [TDLA (triple DLA) clusters] and so forth. We then get a sequence of DLA clusters starting from a SDLA cluster. If our expectation about the anisotropy in DLA clusters is right, then the more enhanced anisotropy should appear as the number of hierarchical repetitions of the (ac) rule with steps (i)–(iv) increases.

Now let us discuss our simulation results. Since we want to grow DDLA clusters of a small scale, we have used five SDLA clusters which have 15 000 particles grown by the (ac) rule. Such SDLA clusters are more or less isotropic with the fractal dimension $D \simeq 1.7$ [7]. Let us first discuss the results for DDLA clusters which have been grown by the (ac) rule with steps (i)–(iv), i.e., by the normal growth rule. We have grown four DDLA clusters with 3000 particles on each of such five SDLA clusters and let us call these 20 DDLA clusters (ac) DDLA clusters. In Fig. 2, we have shown two of four DDLA clusters grown on a SDLA cluster. The (ac) DDLA cluster in Fig. 2(a) has grown mainly along axes of the square lattice and we can see the expected anisotropy well. The DDLA cluster in Fig. 2(b) has a main branch grown along an off-axis direction. We have grown several DLA clusters (i.e., TDLA clusters) on the DDLA cluster in Fig. 2(b) by (ac) rule, and one such TDLA cluster with more enhanced anisotropy is shown in Fig. 2(c). By growing (ac) TDLA clusters on (ac) DDLA clusters we have seen the inclination that the anisotropy gets more and more enhanced if the number of the hierarchical repetitions of the (ac) rule increases as in Fig. 2(c).

A quantitative analysis of the anisotropy in (ac) DDLA clusters is shown in Fig. 3. $\langle N(\theta)\rangle$ in Fig. 3 is defined as what follows. If the coordinate of ith particle in a cluster is (x_i, y_i) in the coordinate system with the origin at the seed and whose x axis and y axis are the same as two lattice axes through the seed, then one can assign the angle θ_i to the particle as $\theta_i = \arctan(y_i/x_i)$. Then $N(\theta)\,d\theta$ is the number of particles in the sector between the angle θ and $\theta + d\theta$. $N(\theta)$ of course satisfies the relation

$$\int_0^{2\pi} N(\theta) d\theta = N_{\text{tot}} , \tag{1}$$

where N_{tot} is the total number of particles in a cluster. In Fig. 3 $\langle N(\theta)\rangle$ means the average of $N(\theta)$ over (ac) DDLA clusters. We see clearly that $\langle N(\theta)\rangle$ has four major peaks and the peaks are located approximately at 0°, 90°, 180°, and 270°. In Fig. 4 we have also displayed $\langle N(\theta)\rangle$ for the SDLA clusters with 15 000 particles which we have used as base clusters for DDLA clusters. In Fig. 4 we cannot find such major peaks as clearly as in Fig. 3. To compare (ac) DDLA clusters to differently grown DDLA clusters, we have also grown three more kinds of DDLA clusters with 3000 particles on the same five SDLA clusters by the (acd) rule, the (bc) rule, and the (bcd) rule. The angular distributions of the particles, $\langle N(\theta)\rangle$'s, for the three kinds of 20 DDLA clusters by (ac), (bc), and (bcd) rules are displayed in Fig. 5. We cannot see four peaks in Fig. 5 as clearly as in Fig. 3. However among the three curves in Fig. 5 that for the (acd) rule manifests the anisotropy more clearly than the other two curves. These results imply that the hopping directions of RW's are a more important part of the growth rule for making the clusters anisotropic than the choice

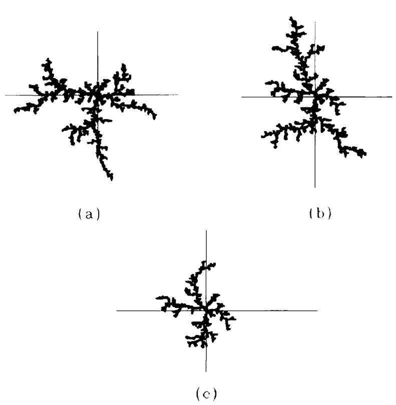

FIG. 2. (a) An (ac) DDLA cluster on a SDLA cluster in which we can see the expected anisotropy rather well. (Two straight lines are the lattice axes through the seed.) (b) Another DDLA cluster which has a main branch grown off the lattice axes. (c) A TDLA cluster on the DDLA cluster in (b). We can see the shortened off-axis branches.

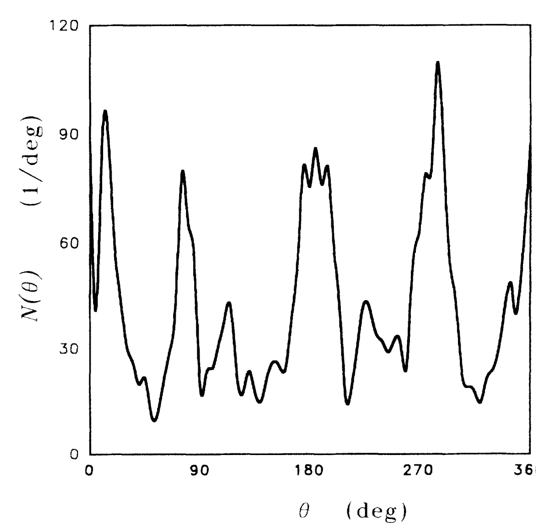

FIG. 3. The relation of $\langle N(\theta)\rangle$ to angle θ for (ac) DDLA clusters.

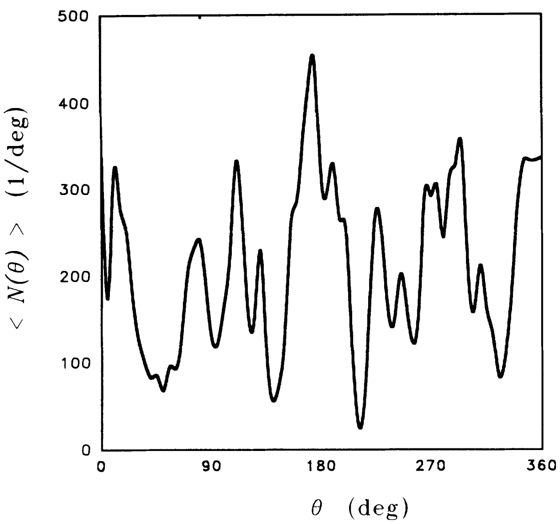

FIG. 4. The relation of $\langle N(\theta)\rangle$ to angle θ for base SDLA clusters.

of growth sites, because clusters by the (ac) rule and those by the (acd) rule show the anisotropy more clearly than clusters by the (bc) and the (bcd) rule.

The length l and the width w of main branches of a cluster can be define through the relations

$$l=\sum_i \max(|x_i|,|y_i|)/N \equiv AN^{\nu_\parallel} \tag{2}$$

and

$$w=\sum_i \min(|x_i|,|y_i|)/N \equiv BN^{\nu_\perp}, \tag{3}$$

where N is the number of particles in the cluster during the growth. The results for the dependence of the averaged $\langle l\rangle$ and $\langle w\rangle$ over (ac) DDLA clusters are displayed in Fig. 6. The estimates for the exponents $\nu_\parallel$ and $\nu_\perp$ are $\nu_\parallel=0.65\pm0.01\simeq\frac{2}{3}$ and $\nu_\perp=0.56\pm0.02$. Comparing these estimates to those for ordinary SDLA clusters with the size of 4×10^6 [7], the envelopes of (ac) DDLA clusters are diamond shaped which we can see only in SDLA clusters of the size of 10^5 or more. We have also analyzed the relations of l's to N and the relations of w's to N for DDLA clusters by (bc), by (bcd), and by (acd) rules and have found $\nu_\parallel\simeq\frac{2}{3}$ for these clusters, but data for w's did not satisfy the power law $w=BN^{\nu_\perp}$ well. The analyses of $\langle l\rangle$ and $\langle w\rangle$ imply that the anisotropy in (ac) DDLA clusters of a small scale is eminent, but the anisotropy cannot be seen clearly in the other kinds of DDLA clusters.

The final conclusions are briefly as follows. The growth of DDLA clusters is a good probe of SDLA clusters, because we can see such an anisotropy quite well in (ac) DDLA clusters of 3000 particles as seen in SDLA clusters at a much larger scale. By comparing differently grown DDLA clusters we have also found that the hopping directions of RW's are the more important part of the growth rule for the anisotropy than the choice of growth sites.

We now want to give final discussions. The first discussion is a comparison of our growth model to noise reduced (or anisotropy enhanced) models [5,13–15]. In order to add a site to a DDLA cluster two walkers have to arrive at the same site, one when the SDLA is grown and the second when DDLA is generated. In the noise-reduced model with noise-reduction parameter m a site must be visited by RW's at least m times to be added to the cluster. Thus two growing methods have nearly the same influence on the growth of a site and the results of both seem to be qualitatively the same. One of the main merits of our model is that on a given SDLA cluster several different kinds of DDLA clusters can be grown by applying different growth rules and then we can under-

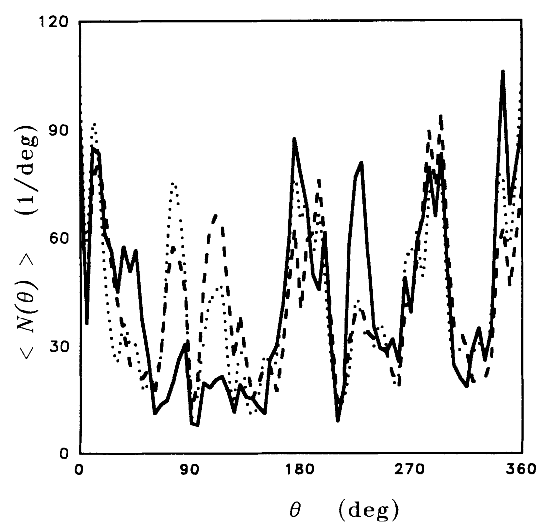

FIG. 5. The relations of $\langle N(\theta)\rangle$'s to angle θ for different DDLA clusters. The dotted curve is for clusters grown by the (acd) rule. The normal curve is for those by the (bc) rule. The dashed curve is for those by the (bcd) rule.

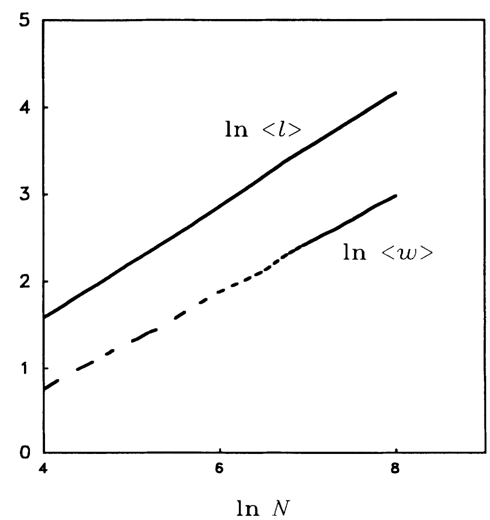

FIG. 6. The relation of the averaged length $\langle l\rangle$ and width $\langle w\rangle$ over (ac) DDLA clusters to the particle number N.

stand which part of growth rule is important to make the resulting cluster anisotropic. In contrast, it is difficult to grow several different kinds of clusters in a noise-reduced model. Even though the relationship between SDLA and DDLA has not been well established on a theoretical basis, a partial clue to understanding the physical origin of large-scaled SDLA clusters is given by the results from the growth of the several differently grown DDLA clusters of a small scale.

One might doubt why the size of the DDLA clusters in this paper is confined to the size of 3000 particles. There are two crucial reasons for this. One is related to properties of SDLA clusters which we have used as base clusters. If the particles of SDLA clusters are about 10^5 then the envelopes of the clusters are diamond shaped [7], and thus to study the anisotropy in a DDLA cluster grown on such large SDLA clusters is meaningless. For our motivation the maximum size of SDLA clusters should be around 50 000. The second one is based on the fact that the size of a DDLA cluster should be much smaller than that of the SDLA cluster. When the size of a DDLA cluster is comparable to that of the SDLA cluster, the shape of the DDLA cluster must be nearly the same as the SDLA cluster itself because of steps (iii) and (iv), and thus to study the DDLA cluster of this size is physically meaningless. Another reason for the existence of the upper bound for the size of a DDLA cluster is that after the tip of a small branch of SDLA cluster is occupied by a particle in a DDLA cluster, it is unnatural to grow the DDLA cluster further. Our choice is to grow DDLA clusters of 3000 particles on SDLA clusters of 15 000 particles. If one chooses the SDLA clusters of 50 000 particles which we think are the largest SDLA clusters for our purpose, the maximum cluster size of DDLA clusters which satisfies the above-mentioned reasons is less than 10 000. The properties of DDLA clusters of that size do not deviate significantly from those of our clusters.

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